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NUMERICAL INVESTIGATION OF THE FORMULAE  
FOR THE ELIMINATION OF  
GEOCENTRIC AND BARYCENTRIC PARALLAX  
IN  
PROF. LEUSCHNER'S SHORT METHOD.

*by Sarah Del. Morgan*

This Thesis Submitted in Partial Fulfillment  
of the Requirements for the Degree  
of  
Master of Arts  
in the  
University of California.  
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Professor Leuschner has made a theoretical investigation<sup>1</sup> of the effect of parallax on an orbit and has given formulae for the elimination of the parallax which are independent of the geocentric distances. The purpose of this investigation is to make numerical tests of these formulae.

The parallax to be considered is twofold: geocentric and barycentric. Geocentric parallax is the angle at the body subtended by the radius of the earth drawn to the point of observation. Barycentric parallax is the angle at the body subtended by the distance from the center of the earth to the center of mass of earth and moon. The barycentric place enters into the problem by virtue of the fact that the equations of apparent motion of the Sun

$$X'' = -\frac{X}{R}; \quad Y'' = -\frac{Y}{R}; \quad Z'' = -\frac{Z}{R}$$

by means of which  $X''$ ,  $Y''$  and  $Z''$  are eliminated from

$$\xi'' = x'' + X''; \quad \eta'' = y'' + Y''; \quad \zeta'' = z'' + Z''$$

1): Pub. of L. O., Vol. VII.



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of the effect of parallax on an orbit and has given formulas for the elimination of the parallax which are independent of the geocentric distances. The purpose of this investigation is to make numerical tests of these formulas.

The parallax to be considered is twofold: geocentric

and heliocentric. Geocentric parallax is the angle at the body subtended by the radius of the earth drawn to the point of observation. Heliocentric parallax is the angle at the body subtended by the distance from the center of the earth to the

center of mass of earth and moon. The heliocentric place enters into the problem by virtue of the fact that the equations of

apparent motion of the Sun

$$X = \frac{x}{K}, Y = \frac{y}{K}, Z = \frac{z}{K}$$

by means of which "x" and "y" are eliminated from

$$x' = x'' + X, y' = y'' + Y, z' = z'' + Z$$

1) Pub. of A. O., Vol. VII.



~~must be referred to the center of mass of the earth and moon~~  
~~as origin in order that the problem may be a two-body problem.~~  
*so that  $\xi''$ ,  $\eta''$ ,  $\zeta''$  must also be referred to this center of mass.*

Both of these parallaxes consist of two parts - a systematic and a variable part. The variable part is equal to the differences of the parallax corrections for the three dates of observation.

For a partial elimination of parallax only the systematic part is eliminated by applying the following corrections to the Solar Coordinates for the Middle Date:

$$\begin{aligned}\Delta X &= p_{\alpha} \Delta \cos \delta \sin \alpha + p_{\delta} \Delta \cos \alpha \sin \delta \\ (a) \quad \Delta Y &= p_{\delta} \Delta \sin \alpha \sin \delta - p_{\alpha} \Delta \cos \delta \cos \alpha \\ \Delta Z &= -p_{\delta} \Delta \cos \delta\end{aligned}$$

For a complete elimination of the parallax two corrections are applied to the solar velocities in addition to the corrections given by (a). The first correction is  $\Delta X'_0, \Delta Y'_0, \Delta Z'_0$  which is obtained from the  $\Delta X, \Delta Y, \Delta Z$  of the three dates by a formula similar to that given for  $\alpha'_0$  in the Publications of the Lick Observatory, Vol. VII.

$$\begin{aligned}\Delta X'_0 &= \frac{\Delta X'_1(t_0 - t_1) + \Delta X'_{11}(t_{11} - t_0)}{t_{11} - t_1} \\ (b) \quad \Delta Y'_0 &= \frac{\Delta Y'_1(t_0 - t_1) + \Delta Y'_{11}(t_{11} - t_0)}{t_{11} - t_1} \\ \Delta Z'_0 &= \frac{\Delta Z'_1(t_0 - t_1) + \Delta Z'_{11}(t_{11} - t_0)}{t_{11} - t_1}\end{aligned}$$



as origin is referred to the center of mass of the earth and moon  
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Both of these parallaxes consist of two parts - a systematic  
 and a variable part. The variable part is equal to the differ-  
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 servation.

For a partial elimination of parallax only the sys-  
 tematic part is eliminated by applying the following corrections  
 to the Solar Coordinates for the Middle Date:

$$\begin{aligned} \Delta X &= \frac{1}{2} \Delta \cos \alpha + \frac{1}{2} \Delta \cos \alpha \sin \delta \\ \Delta Y &= \frac{1}{2} \Delta \sin \alpha \sin \delta - \frac{1}{2} \Delta \cos \alpha \cos \delta \\ \Delta Z &= -\frac{1}{2} \Delta \cos \delta \end{aligned} \quad (a)$$

For a complete elimination of the parallax two correc-  
 tions are applied to the solar velocities in addition to the  
 corrections given by (a). The first correction is  $\Delta X', \Delta Y', \Delta Z'$   
 which is obtained from the  $\Delta X, \Delta Y, \Delta Z$  of the three dates by  
 a formula similar to that given for  $\alpha'$  in the definition of  
 the Iick Observatory, Vol. VII.

$$\begin{aligned} \Delta X'_0 &= \frac{\Delta X(t_0 - t_1) + \Delta X(t_0 - t_2)}{t_0 - t_1} \\ \Delta Y'_0 &= \frac{\Delta Y(t_0 - t_1) + \Delta Y(t_0 - t_2)}{t_0 - t_1} \\ \Delta Z'_0 &= \frac{\Delta Z(t_0 - t_1) + \Delta Z(t_0 - t_2)}{t_0 - t_1} \end{aligned} \quad (b)$$



The second correction to be applied to the solar velocities is one involving a quantity  $\beta$  which is defined as follows:

$$\beta = \frac{-\xi \sin(a - \alpha_1) \left[1 - \frac{\tan P}{\tan P'}\right]}{\alpha_0' - \cos \delta \tan P \tan \delta_0'}$$

These formulas are derived from the formulas for any correction  $s$ ,  $a$ ,  $P$  and  $P'$  are auxiliary quantities depending on the solar coordinates and their corrections, the comet's right ascension and declination and the intervals. This correction depends, in the main, upon the second differences of the  $\Delta X, \Delta Y, \Delta Z$  and is necessary to refer ~~the expression (a)~~  <sup>$\xi'' \eta'' \zeta''$</sup>  to the center of the earth. The corrected solar velocities become

$$\begin{aligned} (X_0') &= X_0' + \Delta X_0' + \beta \cos \alpha_1 \\ (a) \quad (Y_0') &= Y_0' + \Delta Y_0' + \beta \sin \alpha_1 \\ (Z_0') &= Z_0' + \Delta Z_0' + \beta \tan \delta_1 \end{aligned}$$

The solar coordinates and velocities used above refer to the center of the earth as origin. Corrections must also be applied to the solar coordinates and velocities in order that they may be referred to the center of mass as origin. Let

$\Delta_1 X, \Delta_1 Y, \Delta_1 Z$  be the corrections necessary for elimination of the geocentric parallax of the observed body (formula (a));

$\Delta_2 X, \Delta_2 Y, \Delta_2 Z$  for the barycentric parallax of the body. These

Truett, L. J., Vol. VII, part 7, page 21, formulas (15).



The second correction to be applied to the solar velocities is one involving a quantity  $\theta$  which is defined as follows:

$$\theta = \frac{-\frac{1}{2} \sin(\alpha - \alpha') \left[ 1 - \frac{\tan^2 \delta}{\tan^2 \delta'} \right]}{\sin \delta - \sin \delta' \tan \delta \tan \delta'}$$

$\alpha$ ,  $\alpha'$ ,  $\delta$  and  $\delta'$  are auxiliary quantities depending on the solar coordinates and their corrections, the comet's right ascension and declination and the intervals. This correction depends, in the main, upon the second differences of the  $\Delta X, \Delta Y, \Delta Z$  and is necessary to refer the observations to the center of the earth. The corrected solar velocities become

$$\begin{aligned} X_0' &= X_0 + \Delta X_0 + \theta \cos \alpha_0 \\ Y_0' &= Y_0 + \Delta Y_0 + \theta \sin \alpha_0 \\ Z_0' &= Z_0 + \Delta Z_0 + \theta \tan \alpha_0 \end{aligned} \quad (4)$$

The solar coordinates and velocities used above refer to the center of the earth as origin. Corrections must also be applied to the solar coordinates and velocities in order that they may be referred to the center of mass as origin. Let  $\Delta X, \Delta Y, \Delta Z$  be the corrections necessary for elimination of the geocentric parallax of the observed body (Formula (2));  $\Delta X, \Delta Y, \Delta Z$  for the barycentric parallax of the body. These



are also obtained from formulae (a) with the exception that the barycentric parallax factors are used. These factors are

$$p_2^m = d, \frac{\cos \delta_1}{\cos \delta} \sin(\alpha - \alpha_1)$$

$$p_5^m = d_1 \{ \sin \delta \cos \delta_1 \cos(\alpha - \alpha_1) - \cos \delta \sin \delta_1 \}$$

These formulae are derived from the formulae for any correction to  $\alpha$  and  $\delta$  \*.  $\alpha_1$  and  $\delta_1$  are the moon's right ascension and declination at the dates of the observations.

$$\Delta_2 X = p_2^m \cos \delta \sin \alpha + p_5^m \cos \alpha \sin \delta$$

$$(4) \quad \Delta_2 Y = p_5^m \sin \alpha \sin \delta - p_2^m \cos \delta \cos \alpha$$

$$\Delta_2 Z = -p_5^m \cos \delta$$

Corrections  $\Delta_2 X, \Delta_2 Y, \Delta_2 Z$  for barycentric parallax must also be applied to the Solar coordinates. The Solar coordinates  $X, Y$  and  $Z$  referred to the center of mass as origin are equal to the coordinates referred to the center of earth plus the coordinates of the center of the earth referred to the center of mass. Let  $d$  be the distance of the center of mass from the center of the earth. Let  $\alpha_1$  and  $\delta_1$  be the coordinates of the moon and as the center of mass lies in the line joining the center of the moon with the center of the earth,

\*Publ. L. O. Vol. VII, part 7, page 11, formulae (15).



are also obtained from formulae (a) with the exception that the barycentric parallax factors are used. These factors are

$$p_1^{\text{par}} = d \cdot \frac{\sin(\alpha - \alpha_1)}{\sin \delta_1} \quad p_2^{\text{par}} = d \cdot \frac{\sin(\alpha - \alpha_2)}{\sin \delta_2}$$

These formulae are derived from the formulae for any correction to  $\alpha$  and  $\delta$ , and the moon's right ascension and declination at the dates of the observations.

$$\begin{aligned} \Delta X &= p_1^{\text{par}} \cos \alpha + p_2^{\text{par}} \cos \alpha_1 \\ \Delta Y &= p_1^{\text{par}} \sin \alpha + p_2^{\text{par}} \sin \alpha_1 \\ \Delta Z &= -p_1^{\text{par}} \sin \delta - p_2^{\text{par}} \sin \delta_1 \end{aligned} \quad (u)$$

Corrections  $\Delta X, \Delta Y, \Delta Z$  for barycentric parallax must also be applied to the Solar coordinates. The Solar coordinates  $X, Y$  and  $Z$  referred to the center of mass as origin are equal to the coordinates referred to the center of earth plus the coordinates of the center of the earth referred to the center of mass. Let  $d$  be the distance of the center of mass from the center of the earth. Let  $\alpha, \delta$  be the coordinates of the moon and  $\alpha_1, \delta_1$  the center of mass lies in the line joining the center of the moon with the center of the earth.



In order that the solar velocities be referred to the center of mass the rectangular geocentric coordinates of the center of mass are

$$\begin{aligned} x &= d_1 \cos \delta_1 \cos \alpha_1 \\ (f) \quad y &= d_1 \cos \delta_1 \sin \alpha_1 \\ z &= d_1 \sin \delta_1 \end{aligned}$$

The coordinates of the center of the earth referred to the center of mass as origin are equal to  $(\frac{f}{g})$  with opposite signs. Therefore

$$\begin{aligned} \Delta_3 X &= -d_1 \cos \delta_1 \cos \alpha_1 \\ (g) \quad \Delta_3 Y &= -d_1 \cos \delta_1 \sin \alpha_1 \\ \Delta_3 Z &= -d_1 \sin \delta_1 \end{aligned}$$

The value of the distance  $d_1$  is obtained from

$$d_1 = \frac{\pi}{\pi_1} \frac{\mu_1}{1 + \mu_1} \quad (1909.0) \quad (1909.0)$$

where  $\pi$  and  $\pi_1$  are the mean equatorial horizontal parallaxes of the sun and moon.  $\mu_1 = \frac{m}{M} = \frac{\text{mass of the moon}}{\text{mass of the earth}}$ .

Therefore  $d_1 = 6''.4373$ .

The corrected solar coordinates corresponding to the elimination of the geocentric and barycentric parallax in the coordinates and velocities, these being the first coordinates become

$$\begin{aligned} (X) &= X + \Delta_1 X + \Delta_2 X + \Delta_3 X \\ (Y) &= Y + \Delta_1 Y + \Delta_2 Y + \Delta_3 Y \\ (Z) &= Z + \Delta_1 Z + \Delta_2 Z + \Delta_3 Z \end{aligned}$$

Orbit 11 - Geocentric parallax fully determined using Dr.



the rectangular geocentric coordinates of the center of mass

are

$$\begin{aligned} x &= d \cos \delta \cos \alpha, \\ y &= d \cos \delta \sin \alpha, \\ z &= d \sin \delta, \end{aligned} \quad (1)$$

The coordinates of the center of the earth referred to the center of mass as origin are equal to (2) with opposite signs.

Therefore

$$\begin{aligned} \Delta_x X &= -d \cos \delta \cos \alpha, \\ \Delta_y Y &= -d \cos \delta \sin \alpha, \\ \Delta_z Z &= -d \sin \delta, \end{aligned} \quad (2)$$

The value of the distance  $d$  is obtained from

$$d = \frac{\pi}{\pi + \mu} \frac{m}{m+1}$$

where  $\pi$  and  $\mu$  are the mean equatorial horizontal paral-

laxes of the sun and moon.  $\mu = \frac{m}{M} = \frac{\text{mass of the moon}}{\text{mass of the earth}}$

Therefore  $d = 0.0025$ .

The corrected solar coordinates corresponding to the

elimination of the geocentric and barycentric parallax in the

coordinates become

$$\begin{aligned} X &= x + \Delta_x X + \Delta_x x \\ Y &= y + \Delta_y Y + \Delta_y y \\ Z &= z + \Delta_z Z + \Delta_z z \end{aligned} \quad (3)$$



In order that the solar velocities be referred to the center of mass corrections  $\Delta_3 X'_0, \Delta_3 Y'_0, \Delta_3 Z'_0$  derived from formulae (b) using  $\Delta_3 X, \Delta_3 Y, \Delta_3 Z$  from (a) for three dates are applied.

It is evident of course that the complete elimination of geocentric and barycentric parallax should give the same results as if the observations had been corrected beforehand for parallax on the basis of the known value of  $\rho$  (the geocentric distance) and the proper parallax factors.

In order to test the effect of the parallax corrections on an orbit and in order to test the formulae for the elimination of parallax the following orbits have been computed and compared. All of these orbits are based on the same three observations of comet a 1909 (Daniel).

	1909 Gr. M. T.	$\alpha$ (1909.0)	$\delta$ (1909.0)
1.	June 16.5306	25° 28' 38".00	+ 29° 58' 25".00
11.	June 18.9809	27 12 29 .00	+ 33 26 22 .00
111.	June 21.9659	29 27 51 .00	+ 37 25 17 .00

The computation has been carried only to the heliocentric coordinates and velocities, these being the first comparable quantities in the orbits.

Orbit 1 - Geocentric parallax partially eliminated by correcting Solar coordinates of the middle place - computed by Dr. Crawford.

Orbit 11 - Geocentric parallax fully determined using Dr.



In order that the solar velocities be referred to the center of mass corrections  $\Delta X, \Delta Y, \Delta Z$  derived from formulae (a) using  $\Delta X, \Delta Y, \Delta Z$  from (b) for three cases are applied. It is evident of course that the complete elimination of eccentric and perihelionic parallax should give the same results as if the observations had been corrected beforehand for parallax on the basis of the known value of  $p$  (the eccentric distance) and the proper parallax factors. In order to test the effect of the parallax corrections on an orbit and in order to test the formulae for the elimination of parallax the following orbits have been computed and compared. All of these orbits are based on the same three observations of comet 1909 (Daniel).

	1909 Gr. W. T.	$\Delta$ (1909.0)	$\delta$ (1909.0)
I.	June 15.5305	82° 23' 38".00	+ 28° 28' 25".00
II.	June 18.9803	27 12 27.00	+ 33 22 22.00
III.	June 21.9823	29 24 51.00	+ 37 22 17.00

The computation has been carried only to the heliocentric coordinates and velocities, these being the first comparable quantities in the orbits.

Orbit I - Eccentric parallax partially eliminated by correcting Solar coordinates at the middle place - computed by Dr. Brewster.  
Orbit II - Eccentric parallax fully determined using Dr.



Crawford's parallax factors and geocentric distances.

Orbit III - Geocentric parallax completely eliminated by the formulae.

Orbit IV - Parallax entirely neglected.

Orbit V - Geocentric and barycentric parallax completely eliminated by the formulae.

Orbit VI - Observations corrected for geocentric and barycentric parallax.

Details of the computation:-

Orbit I - This orbit was computed by Professor Crawford and published in Lick Observatory Bulletin Number 159. The right ascensions and declinations were used as given above without change. The Solar coordinates for all three dates and the Solar velocities for the middle date were interpolated from the Nautical Almanac -

		X	Y	Z
1	June 16.5306	+ 0.0854344	+ 0.9288751	+ 0.4029449
11	June 18.9809	+ 0.0440423	+ 0.9314663	+ 0.4040708
111	June 21.9659	- 0.0064722	+ 0.9324818	+ 0.4045127
		X'	Y'	Z'
11	June 18.9809	9.992630 <sub>n</sub>	8.630115	8.268346

The geocentric parallax is partially eliminated for the second dates by formulae (a).



Gravford's parallax factors and geocentric distances.

Orbit III - Geocentric parallax completely eliminated by

the formulas.

Orbit IV - Parallax entirely neglected.

Orbit V - Geocentric and heliocentric parallax completely

eliminated by the formulas.

Orbit VI - Observations corrected for geocentric and helio-

centric parallax.

Details of the computation:-

Orbit I - This orbit was computed by Professor Gravford

and published in Lick Observatory Bulletin Number 152. The

right ascensions and declinations were used as given above

without change. The Solar coordinates for all three dates and

the Solar velocities for the middle date were interpolated

from the Nautical Almanac -

	X	Y	Z
I June 18.5306	+ 0.0004344	+ 0.7293781	+ 0.4033449
II June 18.5309	+ 0.0460423	+ 0.9314683	+ 0.4040705
III June 21.5339	- 0.0064723	+ 0.9334313	+ 0.4045137
II June 18.5309	2.9993830	8.656118	8.883348

The geocentric parallax is partially eliminated for the

second dates by formulas (a).



$$\Delta X_0 = 0.0000073$$

$$\Delta Y_0 = 0.0000320$$

$$\Delta Z_0 = 0.0000123$$

These corrections are applied to X, Y and Z of the middle place. Then:

$$X_0 + 0.0440350$$

$$Y_0 + 0.9314983$$

$$Z_0 + 0.4040585$$

Orbit 11 - The right ascensions and declinations at the three dates were corrected for geocentric parallax on the basis of the geocentric distances derived in Orbit 1. The parallax corrections are as follows:-

Orbit IV - The right ascensions 11 declinations 111 as in

Orbit 1 and the  $\delta'' = 7''.78$   $\delta'' = 8''.18$   $\delta'' = 8''.65$

Orbit V - The  $\rho'' + 5.96$   $\rho'' + 3.18$   $\rho'' + 3.14$  taken

The corrected observations are:-

corrected for their heliocentric parallax and the heliocentric

and barycentric  $\alpha = 25^\circ 28' 30''.22$   $27^\circ 12' 20''.82$   $29^\circ 27' 42''.35$

by (b).  $\delta + 29 58 30 .96 + 33 26 25 .18 + 37 25 20 .14$

These were made the basis of the calculation together with the Solar Coordinates X, Y, Z as interpolated from the Ephemeris.

Orbit 111 - Based on the same values of  $\alpha$  and  $\delta$  as in Orbit 1. The Solar coordinates for the middle date corrected as in Orbit 1 in addition  $\Delta X, \Delta Y, \Delta Z$  were computed for the first and third dates. The three sets of corrections are as follows:

$\Delta X = 188$   $\Delta Y = 249$   $\Delta Z = 72$

$\Delta X = 89$   $\Delta Y = 269$   $\Delta Z = 132$







$\Delta_1 X_1 - 0.0000010$	$\Delta_1 Y_1 + 0.0000335$	$\Delta_1 Z_1 - 0.0000235$
$\Delta_1 X_0 - 73$	$\Delta_1 Y_0 + 320$	$\Delta_1 Z_0 - 123$
$\Delta_1 X_{11} - 82$	$\Delta_1 Y_{11} + 332$	$\Delta_1 Z_{11} - 120$

From these their velocities by formulae (b) are computed:

$$\log \Delta X_0' = 5.9542, \quad \log \Delta Y_0' = 4.9547, \quad \log \Delta Z_0' = 6.1719$$

In addition  $\beta$  was computed by formula (c). The resulting values of  $X_0'$ ,  $Y_0'$  and  $Z_0'$  from (d) are as follows:

$$X_0' = 9.992280, \quad Y_0' = 8.634617, \quad Z_0' = 8.286768$$

Orbit IV - The right ascensions and declinations as in Orbit I and the Solar Coordinates uncorrected.

Orbit V - The right ascensions and declinations are taken as in Orbit I. The solar coordinates for the three dates are corrected for their barycentric parallax and the geocentric and barycentric parallaxes in the observed places eliminated by (h). The corrections applied to the Solar Coordinates are:

1.

$\Delta_1 X_1 - 0.0000010$	$\Delta_1 Y_1 + 0.0000335$	$\Delta_1 Z_1 - 0.0000235$
$\Delta_2 X_1 - 91$	$\Delta_2 Y_1 - 189$	$\Delta_2 Z_1 - 2$
$\Delta_3 X_1 - 90$	$\Delta_3 Y_1 - 275$	$\Delta_3 Z_1 + 117$

11.

$\Delta_1 X_0 - 73$	$\Delta_1 Y_0 + 320$	$\Delta_1 Z_0 - 123$
$\Delta_2 X_0 - 168$	$\Delta_2 Y_0 - 269$	$\Delta_2 Z_0 - 71$
$\Delta_3 X_0 + 86$	$\Delta_3 Y_0 - 269$	$\Delta_3 Z_0 + 132$



$\Delta X' - 0.0000010$	$\Delta Y' + 0.0000035$	$\Delta Z' - 0.0000035$
$\Delta X'' - 73$	$\Delta Y'' + 320$	$\Delta Z'' - 185$
$\Delta X''' - 82$	$\Delta Y''' + 332$	$\Delta Z''' - 120$

From these their velocities by formula (a) are computed:

$$\Delta X' = 0.0000010, \Delta Y' = 0.0000035, \Delta Z' = 0.0000035$$

In addition  $\Delta$  was computed by formula (a). The resulting

values of  $X'$ ,  $Y'$  and  $Z'$  from (d) are as follows:

$$X' = 0.0000010, Y' = 0.0000035, Z' = 0.0000035$$

Orbit IV - The right ascensions and declinations are in

Orbit I and the Solar coordinates are corrected.

Orbit V - The right ascensions and declinations are taken

as in Orbit I. The solar coordinates for the three cases are

corrected for their heliocentric parallax and the geocentric

and heliocentric parallaxes in the observed places eliminated

by (h). The corrections applied to the Solar coordinates are:

I.

$\Delta X' - 0.0000010$	$\Delta Y' + 0.0000035$	$\Delta Z' - 0.0000035$
$\Delta X'' - 91$	$\Delta Y'' - 182$	$\Delta Z'' - 2$
$\Delta X''' - 40$	$\Delta Y''' - 275$	$\Delta Z''' + 117$

II.

$\Delta X' - 73$	$\Delta Y' + 320$	$\Delta Z' - 185$
$\Delta X'' - 188$	$\Delta Y'' - 389$	$\Delta Z'' - 41$
$\Delta X''' + 80$	$\Delta Y''' - 389$	$\Delta Z''' + 122$



111.

$\Delta_1 X_{\text{u}} = 0.0000082$	$\Delta_1 Y_{\text{u}} + 0.0000332$	$\Delta_1 Z_{\text{u}} = 0.0000120$
$\Delta_2 X_{\text{u}} = 214$	$\Delta_2 Y_{\text{u}} = 276$	$\Delta_2 Z_{\text{u}} = 129$
$\Delta_3 X_{\text{u}} + 251$	$\Delta_3 Y_{\text{u}} = 158$	$\Delta_3 Z_{\text{u}} + 97$

The solar coordinates become:

$X_1 + 0.0854153$	$Y_1 + 0.9288622$	$Z_1 + 0.4029329$
$X_0 + 0.0440268$	$Y_0 + 0.9314445$	$Z_0 + 0.4040646$
$X_{\text{u}} = 0.0064767$	$Y_{\text{u}} + 0.9324716$	$Z_{\text{u}} + 0.4044975$

The solar velocities are referred to barycentric place by means of the  $\Delta_3 X, \Delta_3 Y, \Delta_3 Z$  given above. Using formula (b).

$\log \Delta X'_0 = 6.5731$	$\log \Delta Y'_0 = 6.0223$	$\log \Delta Z'_0 = 5.0486$
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The Solar velocities which are referred to the center of mass are corrected for geocentric and barycentric parallax using formula (d). applied  $\Delta_1 X, \Delta_2 X; \Delta_1 Y, \Delta_2 Y$  etc

$\log \Delta X'_0 = 6.3630$	$\log \Delta Y'_0 = 6.0770$	$\log \Delta Z'_0 = 4.8886$
$\log \rho \cos \alpha = 6.9824$	$\log \rho \sin \alpha = 6.6934$	$\log \beta \tan \delta = 6.8531$

The solar velocities are as follows:

$(X'_0) 9.992142$	$(Y'_0) 8.634969$	$(Z'_0) 8.284648$
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Orbit VI - The right ascensions and declinations at the three dates are corrected for geocentric and barycentric parallax on the basis of the geocentric distances derived in Orbit 1. The parallax corrections are as follows:



III.

$\Delta X - 0.000003$	$\Delta Y + 0.000023$	$\Delta Z - 0.000010$
$\Delta X - 214$	$\Delta Y - 276$	$\Delta Z - 129$
$\Delta X + 251$	$\Delta Y - 128$	$\Delta Z + 27$

The solar coordinates become:

$X' + 0.000115$	$Y' + 0.000023$	$Z' + 0.000023$
$X' + 0.010026$	$Y' + 0.010026$	$Z' + 0.010026$
$X' - 0.000477$	$Y' + 0.000023$	$Z' + 0.000477$

The solar velocities are referred to barycentric place by means of the  $\Delta X, \Delta Y, \Delta Z$  given above. Using formula (b).

The solar velocities which are referred to the center of mass are corrected for geocentric and barycentric parallax using

formula (d).  $\Delta X, \Delta Y, \Delta Z$

$\Delta X' + 0.0000$	$\Delta Y' + 0.0000$	$\Delta Z' + 0.0000$
$\Delta X' + 0.0000$	$\Delta Y' + 0.0000$	$\Delta Z' + 0.0000$

The solar velocities are as follows:

$(X') 0.000115$	$(Y') 0.000023$	$(Z') 0.000023$
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Orbit VI - The right ascension and declination at the

three dates are corrected for geocentric and barycentric parallax on the basis of the geocentric distances derived in Orbit I. The parallax corrections are as follows:



	1.	II.	III.
$\beta''$	- 13".08	- 15".36	- 15".51
$\beta''$	+ 5 .92	+ 1. 37	+ 0 .25

The corrected observations are

	1.	II.	III.
$\alpha$	25° 28' 24".92	27° 12' 13".64	29° 27' 35".49
$\delta$	+ 29 58 30 .92	+ 33 26 23 .37	+ 37 25 16 .75

The solar coordinates are referred to barycentric place by applying the corrections

$$\Delta X_0 + 0.0000086 \quad \Delta Y_0 - 0.0000269 \quad \Delta Z_0 + 0.0000132$$

The solar coordinates are as follows:

$$X_0 + 0.0440509 \quad Y_0 + 0.9314394 \quad Z_0 + 0.4040840$$

The solar velocities are referred to barycentric place by applying the corrections

$$\log \Delta X_0' \quad 6.5731 \quad \log \Delta Y_0' \quad 6.0223 \quad \log \Delta Z_0' \quad 5.0486$$

The solar velocities then become

$$\log X_0' \quad 9.992465 \quad \log Y_0' \quad 8.631185 \quad \log Z_0' \quad 8.268084$$

The following values of the <sup>comets'</sup> heliocentric coordinates and velocities are obtained for the different orbits.

In numbers.



I.	II.	III.
$\delta'' - 13''.08$	$- 15''.38$	$- 15''.01$
$\delta'' + 8.92$	$+ 1.27$	$+ 0.23$

The corrected observations are

I.	II.	III.
$250^{\circ} 28' 24''.22$	$270^{\circ} 13' 13''.64$	$290^{\circ} 27' 35''.49$
$+ 29.58$	$+ 23.28$	$+ 27.25$
$30.22$	$23.27$	$25.16$

The solar coordinates are referred to barycentric place by

applying the corrections

$$\Delta X_0 + 0.0000036 \quad \Delta Y_0 - 0.0000029 \quad \Delta Z_0 + 0.0000132$$

The solar coordinates are as follows:

$$X_0 + 0.0440809 \quad Y_0 + 0.9314394 \quad Z_0 + 0.4040840$$

The solar velocities are referred to barycentric place by

applying the corrections

$$\log \Delta X_0, 8.5731 \quad \log \Delta Y_0, 8.0283 \quad \log \Delta Z_0, 8.0438$$

The solar velocities then become

$$\log X_0, 9.92222 \quad \log Y_0, 8.93115 \quad \log Z_0, 8.93034$$

The following values of the heliocentric coordinates and

velocities are obtained for the different orbits.

In numbers.



	1.	11.	111.	1V.	V.	V1.
X <sub>0</sub>	+0.668778	+0.669700	+0.669271	+0.668780	+0.669562	+0.669853
Y <sub>0</sub>	-0.565036	-0.564561	-0.564782	-0.565000	-0.564584	-0.564483
Z <sub>0</sub>	+0.125224	+0.125908	+0.125589	+0.125217	+0.125793	+0.125997
X'	+0.446200	+0.443864	+0.445037	+0.446198	-0.443721	+0.443432
Y'	+0.347541	+0.347166	+0.347367	+0.347548	+0.347165	+0.347085
Z'	+1.393368	+1.393785	+1.393542	+1.393387	+1.393897	+1.393841

The following comparisons are of particular interest.

	V1-V	V1-1V	1-1V
X <sub>0</sub>	+ 291	+ 1073	- 2
Y <sub>0</sub>	+ 101	+ 517	+ 36
Z <sub>0</sub>	+ 204	+ 780	+ 7
X'	- 289	- 2766	+ 2
Y'	- 85	- 463	- 7
Z'	+ 216	+ 454	- 19

The differences between Orbit 1V and Orbit V1 show that Orbit 1V is correct only to two places. In the comparison of Orbit 1 and Orbit 1V where the parallax is entirely neglected the differences are so small and at the same time are within the errors of computation that it is quite evident that the partial elimination has no decided effect upon the orbit.

The differences between Orbit V and V1 where parallax



	I.	II.	III.	IV.	V.	VI.
X.	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000
Y.	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000
Z.	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000
X'.	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000
Y'.	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000
Z'.	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000

The following comparisons are of particular interest.

	VI-V	VI-IV	I-IV
X.	+ 201	+ 1078	- 8
Y.	+ 101	+ 819	+ 36
Z.	+ 204	+ 780	+ 7
X'.	- 202	- 1078	+ 8
Y'.	- 101	- 819	- 36
Z'.	+ 210	+ 1084	- 19

The difference between Orbit IV and Orbit VI show that Orbit IV is correct only to two places. In the comparison of Orbit I and Orbit IV where the parallel is entirely neglected the differences are so small and at the same time are within the errors of computation that it is quite evident that the partial elimination has no decided effect upon the orbit.

The differences between Orbit V and VI where parallel



is completely eliminated show that Orbit VI is correct to three places. Orbit V, depending on differential relations, is more accurate than Orbit VI. Therefore the elimination formulae for parallax give an orbit accurate to two more places than the omission of parallax. The gain of two places however is only apparent if the errors of observation are comparable with the variable part of the parallax corrections. The differences between Orbit II and Orbit III, which should check, are, however, within the errors of the computation except in the case of  $X'$  where the largeness of the difference may arise from an accumulated error.

The differences between Orbit V (geocentric and barycentric parallax eliminated) and Orbit VI (geocentric and barycentric parallax fully determined) are of the greatest importance and interest. The differences are within the error of computation and it is seen from them that Orbit VI is correct to four places. This is a gain of two places over Orbit IV. In comparing the differences between Orbit VI and Orbit IV with the differences between Orbit V and Orbit VI it is seen that practically all of the parallax corrections have been taken into account and <sup>the differences</sup> those that remain are within the error of the computation. Therefore for corrt a 1909 Orbit V is the most



is completely eliminated when Orbit VI is correct to three places. Orbit V, depending on differential relations, is more accurate than Orbit VI. Therefore the elimination formulae for parallel give an orbit accurate to two more places than the relation of parallel. The gain of two places however is only apparent if the error of observation are comparable with the variable part of the parallel corrections. The differences between Orbit II and Orbit III, which should check etc, however, within the error of the computation except in the case of  $\chi'$  where the largeness of the difference may arise from an assumed error.

The differences between Orbit V (geocentric) and par-

centric parallel eliminated) and Orbit VI (geocentric and heliocentric parallel fully determined) are of the greatest importance and interest. The differences are within the error of computation and it is seen from them that Orbit VI is correct to four places. This is a gain of two places over Orbit IV. In comparing the differences between Orbit V and Orbit IV with the differences between Orbit V and Orbit VI it is seen that ~~eliminating~~ all of the parallel corrections have been taken into account and hence that ~~the~~ <sup>the</sup> results are within the error of the computation. Therefore for correct a 1909 Orbit V is the most



accurate. The gain in accuracy of two places justifies the added work in the computation.

Although the formulae for the elimination of barycentric and geocentric parallax should be used in comet a 1909, the question presents itself whether this method should be pursued in all cases. As stated at the beginning there is a systematic and a variable part in parallax. The systematic part which alone was taken care of in Orbit I gave but a small correction; the variable part which is the difference between the parallaxes for the different dates of observation and hence the velocities in the parallaxes gives the largest corrections, as is shown in Orbit V. It is evident therefore that if the variable part in the parallaxes is not greater than the error of the observation the corrections would have no value, for if they were smaller the observations would be corrected by a correction to more places than they themselves can be relied upon. The differences in the parallax are greater than the errors of observation only when the comet is rapid moving and fairly near the sun. Then the method of Orbit V is to be pursued. In all other cases the parallax is to be entirely neglected.



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sued. In all other cases the parallax is to be entirely neg-

lected.



BERKELEY, CAL.,

Dec 4, 1909.

Miss Sarah de Camp Morgan  
Students' Observatory.

Dear Miss Morgan,

Orbit V is not quite correct.  
It will be necessary to repeat part of the  
computation, namely p and all the quanti-  
ties depending on same. The equation  
of motion for the Sun is true in the form

$$X'' = \frac{X + \Delta X_3}{(R + \Delta R_3)^3}$$

But we use

$$X'' = \frac{X + \Delta X_1 + \Delta X_2 + \Delta X_3}{(R + \Delta R_1 + \Delta R_2 + \Delta R_3)^3} = \frac{(X)}{(R)^3}$$

which is therefore in error only by  $\Delta X_1 + \Delta X_2$ , and  
 $\Delta R_1 + \Delta R_2$ . The error arising from  $\Delta R_1 + \Delta R_2$  is ineffective  
as it disappears in the ratio  $\frac{C'}{C}$ . The error  $\Delta X_1 + \Delta X_2$   
is taken care of in the term  $\frac{1}{(R)^3}(\Delta X_1 + \Delta X_2)$  of

$$\left( \frac{1}{(R)^3} + \frac{d_x}{C, C_{\infty}} \right) (\Delta X_1 + \Delta X_2)$$

You have used  $(\Delta X)$  in this formulae =  $\Delta X_1 + \Delta X_2 + \Delta X_3$ .  
You should have used  $(\Delta X) = \Delta X_1 + \Delta X_2$ .



BERKELEY, CAL. Oct. 4. 1909

Dear Mr. Hooper  
 Students' Observatory

That  $\bar{v}$  is not quite correct  
 It will be necessary to repeat part of the  
 computation, namely to find all the present  
 the depending on  $\bar{v}$ , the equation  
 of motion for the  $\bar{v}$  is then in the form

$$X'' = \frac{X + \Delta X}{(R + \Delta R)^2}$$

$$X'' = \frac{X + \Delta X + \Delta X}{(R + \Delta R + \Delta R)^2} = \frac{(X)}{(R)^2}$$

which is therefore no more only by  $\Delta X + \Delta X$ , and  
 $\Delta R + \Delta R$ . The error arising from  $\Delta R + \Delta R$  is sufficient  
 as it disappears in the ratio  $\frac{\Delta X}{R}$ . The error  $\Delta X + \Delta X$   
 is taken care of in the term  $\frac{1}{R^2}(\Delta X + \Delta X)$  of

$$\left( \frac{1}{R^2} + \frac{\Delta X}{R^3} \right) (\Delta X + \Delta X)$$

For I have used  $(\Delta X)$  in the formulae =  $\Delta X + \Delta X + \Delta X$   
 you should have used  $(\Delta X) = \Delta X + \Delta X$ .



Miss Hogen - 2 -

UNIVERSITY OF CALIFORNIA  
BERKELEY ASTRONOMICAL DEPARTMENT  
(STUDENTS' OBSERVATORY)

BERKELEY, CAL.,

Similarly  $d_x, d_y, d_z$  should have been computed from the  $\Delta X = \Delta X_1 + \Delta X_2$ , etc. In other words  $\Delta X_3$  does not enter into  $\Delta$  or into  $d_x, d_y, d_z, s, a, d$ , or  $\beta$ . With the exception of this error in  $\beta$  the <sup>first</sup> <sup>on orbit</sup> work is correct & save that you In the second orbit you have (page 5a, etc) you have introduced  $\Delta X_3$  twice. In computing the  $(\Delta X)_0$  from the  $(\Delta X) = \Delta X_1 + \Delta X_2 + \Delta X_3$ , the  $(\Delta X)_3$  is included, so that you should not have revised your work by introducing it a second time into the solar velocities. For  $(X)'_0$  we get then

$$(X)'_0 = X'_0 + (\Delta X_1 + \Delta X_2 + \Delta X_3)'_0 + \beta \cos \alpha, \text{ etc.}$$

Your only error is then in the  $\beta$

Can you rectify the error by Monday, 2 p.m.?  
Orbit vi appears to be correct.

Very sincerely yours,  
A. O. Leuschner



I understand that you should have been contacted  
 from the  $AX = AX_1 + AX_2$  in order to  
 make  $AX$  into not into  $AX_1$  or  $AX_2$   
 up,  $AX_1, 2, AX_2, 3$ . But the exception  
 of the error in  $AX_1$  is correct &  
~~is that you are the second order~~  
 from (page 2) for the introduction  
 $AX_1$  twice. In computing the  $AX_1$  from  
 the  $AX_1 = AX_1 + AX_2$ , the  $AX_1$  is included  
 so that you should not have worked from  
 work by introduction of a second time into  
 the other variables. For  $AX_1$ , we get the  
 $(X_1' - X_2' + (AX_1 + AX_2)' + 3 \text{ error})$   
 from  $AX_1$  as then in the  $3$   
 Can you verify the error of  $AX_1$  from  
 Order it appears to be correct.  
 Very sincerely yours,  
 A. C. Lewis











82 a 1909

Kobold

June 16, 17, 18

T 1909 June 5.35 B.M.T

w 5° 3.54

Ob 305 21.32

i 51 53.62

log q 9.92524

q 0.842

Cranford

June 16 18 21

June 5.17 Gr.M.T

4° 59'

306 19

52 26

0.846

Kobold

16, 20, 24

5.30 B.M.T  
5.26 12.11.17

5° 0' 1'

305 38

52 4

9.92583

0.843

Boss

16, 17, 18 confuz

T 5.12 Gr.M.T.

w 4° 56'

Ob 306 40

i 52 39

log q 9.92815

q 0.848

Kobold II - Cr.

$\Delta T$  + .09 d

$\Delta w$  + 2'

$\Delta Ob$  - 41

$\Delta i$  - 22

$\Delta q$  -.003

Kobold II - B.M.

+ .14 d

+ 5'

- 62

- 35

-.005

Kobold II - I

$\Delta T$  -.05

$\Delta w$  - 2.5'

$\Delta Ob$  + 17

$\Delta i$  + 10

$\Delta q$  .001











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